Mathematics Education Research Journal
https://doi.org/10.1007/s13394-021-00386-7

ORIGINAL ARTICLE

Lower secondary students' encounters with mathematical literacy

Oda Heidi Bolstad[1]

Received: 11 December 2020 / Revised: 26 April 2021 / Accepted: 10 May 2021

Abstract
Worldwide, there has been an increased emphasis on enabling students to recognise the real-world significance of mathematics. Mathematical literacy is a notion used to define the competencies required to meet the demands of life in modern society. In this article, students' encounters with mathematical literacy are investigated. The data comprises interviews with 22 students and observations of 16 mathematics lessons in three grade 9 classes in Norway. The analysis shows that students' encounters with mathematical literacy concern specific mathematical topics and contexts from personal and work life. Students' encounters with ML in school is characterised by an emphasis on developing mathematical knowledge within the school context.

Keywords Mathematical literacy · Numeracy · The theory of objectification · Mathematics education

Introduction

One goal of schooling is for students to acquire knowledge and competences that meet the needs of modern society. Mathematical literacy (ML) is a notion used to define the body of knowledge and competences required to meet the mathematical demands of personal and social life and to participate in society as informed, reflective and contributing citizens (Geiger et al., 2015). ML has many related concepts, such as *numeracy* and *quantitative literacy*. While the term numeracy is more common in the UK, Australia and New Zealand, quantitative literacy and ML are used in the USA (Geiger et al., 2015). Some use these notions synonymously while others distinguish between them. The meaning of numeracy varies from the acquisition of basic arithmetic skills through to richer interpretations related to problem-solving

✉ Oda Heidi Bolstad
bolstado@hivolda.no

1 Volda University College, Volda, Norway

Published online: 16 July 2021

in real-life contexts (Geiger et al., 2015). Quantitative literacy is associated with the requirements connected to the increasing influence of digital technology in society and the forms of thinking and reasoning related to problem-solving in the real world (Steen, 2001). Other perspectives, such as *critical mathematical numeracy* (e.g. Frankenstein, 2010), *mathemacy* (e.g. Skovsmose, 2011) and *matheracy* (e.g. D'Ambrosio, 2007), are concerned with competences for challenging social injustices and for working to promote a more equitable and democratic society. Although these notions do not share the same meaning, their definitions share common features in that they stress awareness of the usefulness of, and ability to use, mathematics in different contexts (Niss & Jablonka, 2014). Typically, they do not discriminate between contexts from employment and everyday life, but the main orientation appears to be toward everyday life and citizenship (Gravemeijer et al., 2017). In this article, ML is conceptualised in a broad way, composed of the others.

ML is one of the educational competencies emphasised by the Organisation for Economic Cooperation and Development (OECD). Curriculum documents around the world have been restructured to include this competence (Stacey & Turner, 2015). For instance, in Norway, ML is one of five basic competences to be developed across school subjects. The Norwegian curriculum describes ML as.

> *applying mathematics in different situations. Being numerate*[1] *means to be able to reason and use mathematical concepts, procedures, facts and tools to solve problems and to describe, explain and predict what will happen. It involves recognizing numeracy in different contexts, asking questions related to mathematics, choosing relevant methods to solve problems and interpreting validity and effect of the results. Furthermore, it involves being able to backtrack to make new choices. Numeracy includes communicating and arguing for choices by interpreting context and working on a problem until it is solved.*
>
> *Numeracy is necessary to arrive at an informed opinion about civic and social issues. Furthermore, it is equally important for personal development and the ability to make appropriate decisions in work and everyday life. (The Norwegian Directorate for Education & Training,* 2012*)*

The current worldwide emphasis on ML is based on the recognition that many students are completing compulsory education without the mathematical skills required in life and work, and that formal mathematics alone is not helping them meet these demands (Liljedahl, 2015). In several places (e.g. Popovic & Lederman, 2015; Vos, 2018), students' view of mathematics is described as detached from reality, and the most frequently asked question in mathematics classrooms is "When will we ever use this?".

Students' perception of mathematics as detached from reality can influence their views of the purpose of mathematics. Some researchers discuss the purposes of

[1] In the English translation of the Norwegian curriculum, the word numeracy is used. However, the PISA framework (OECD 2012) has influenced the description of this competence, and resemblances can be found between the two descriptions. Therefore, in the Norwegian context, and for the purpose of this article, the two notions are taken to mean the same.

mathematics education, but few research studies are concerned with the purpose of mathematics from the students' perspective (Nosrati & Andrews, 2017). Students can contribute with valuable insider perspectives on mathematics education, and there is a need for more research concerning the issue. Such research must also consider the environments in which students learn (Mellin-Olsen, 1981). Situations may occur where students are unable to place the learning situation in any other context than that of school. In such cases, years of mathematics studies may seem to have unclear purposes. Therefore, research needs to consider the nature of students' learning processes, i.e. in terms of teaching, tasks, culture and society.

This article reports from a study that investigates students' mathematics learning regarding connections between mathematics and real life. The aim is to investigate how mathematics classroom activities are connected to students' perception of the contexts in which they need mathematics and their encounters with ML. The research question addressed is:

> What are the characteristics of students' encounters with mathematical literacy?

Students' learning processes are viewed as situated within social, historical and cultural forms of thinking and doing. Therefore, the study is framed within a cultural-historical theory of mathematics teaching and learning. The theoretical perspectives are presented in the following section.

Theoretical Perspectives

The Theory of Objectification

From the works of Vygotsky and Leont'ev, Luis Radford has developed the theory of objectification (TO). TO focuses on how students and teachers produce knowledge against the backdrop of history and culture, and on how they co-produce themselves as subjects in general and subjects in education in particular.

> *The TO is inscribed within an understanding of mathematics education as a political, societal, historical, and cultural endeavor. Such an endeavor aims at the dialectic creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices, and ponder and deliberate on new possibilities of action and thinking. (Radford,* 2016*)*

In TO, knowledge involves potentiality and actuality (Radford, 2015). Potentiality means general and abstract interpretations or actions resulting from cultural and historical ways of thinking and doing, for example, general knowledge about doing calculations. Actuality means that these general interpretations and actions are actualised through something concrete and noticeable, for example, doing a specific calculation. Therefore, in TO, knowledge is not something one possesses but rather something one *encounters*.

Learning happens when the general interpretations are actualised and, in this way, becomes part of the individual's *consciousness*. That is, when, through doing the specific calculation, the individual encounters and becomes aware of the general knowledge about doing calculations. The process of recognising such encounters with knowledge is what Radford terms *processes of objectification*[2] (Radford, 2015).

The process of *subjectification* is closely connected to processes of objectification (Radford, 2016). As the individual becomes more knowledgeable, s/he also changes and develops as a person. Therefore, we are learning because we are becoming, and we are becoming because we are learning. In the research reported here, developing ML is considered as both learning and becoming.

Mathematical literacy

Merrilyn Goos has developed a model designed to capture the richness of current definitions of ML and related concepts (see e.g. Goos et al., 2014). The model has been used in professional development programmes as a tool to plan ML teaching. It is developed and used in the Australian context, but there are several reasons for its relevance in a Norwegian context. First, in Australia, numeracy has been interpreted in a broad sense similar to the OECD definition of ML (Goos et al., 2010). Second, there are similarities between the Norwegian and Australian curricula concerning the Norwegian basic skills and the Australian general capabilities. In both curricula, numeracy is considered a competence to be developed in all subjects, as well as in mathematics specifically. Both countries conduct national tests to assess students' numeracy level. Third, a cluster analysis of the cognitive items in ML from PISA 2003 suggests that the Nordic countries' profiles strongly relate to the profiles of five of the six English-speaking countries participating in PISA (Olsen, 2006). Australia is one of these five countries. Hence, it is reasonable to use the model in the Norwegian context.

The ML model represents the multifaceted nature of ML and involves five interrelated elements: *contexts, mathematical knowledge, tools, dispositions,* and *critical orientation*. The model is presented in Fig. 1.

Contexts are placed at the centre of the model because ML concerns the ability to use mathematics in contexts. Goos et al. (2014) highlight three contexts in which ML is important; personal and social life, work-life and citizenship. Personal and social contexts arise from daily life with the perspective of the individual being central, involving, for instance, personal finance and participation in different leisure activities. Work contexts arise from professional life. People use mathematics in their work, but what they do and how they do it may not be predictable from considerations of general mathematical methods (Noss et al., 2000). Occupations have specific requirements and tasks related to different kinds of mathematical knowledge, such as financial transactions or drug

[2] In the theory of objectification, the notions *objectification* and *subjectification* have specific meanings. It is important to note that the same notions have different meanings when used in other discourses, such as Sfard (2008).

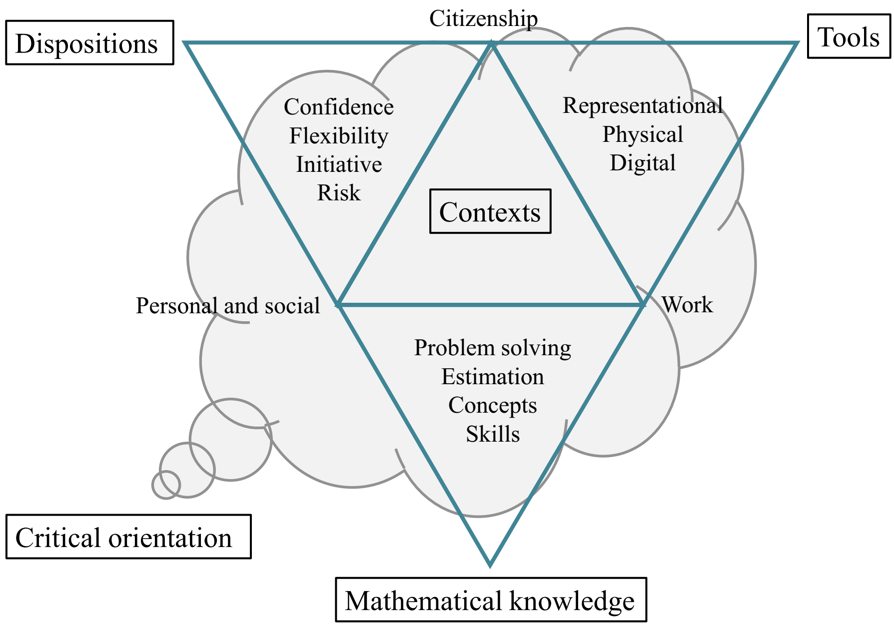

Fig. 1 A model of ML (Goos et al., 2010)

administration. Citizenship concerns societal contexts arising from being a citizen, local, national or global. Every major public issue depends on different types of data, for instance, in understanding a voting system or international economics.

Mathematical knowledge is composed of knowledge of mathematical concepts, procedures, and facts, and using these in problem-solving strategies and estimations to describe, explain and predict. Hence, a part of being mathematically literate involves formal mathematical knowledge and means being able to perform calculations and use procedures and algorithms successfully (Steen et al., 2007).

Tools can be physical items (e.g. measuring instruments or concretes), thinking tools (e.g. different forms of representations such as graphs and algebraic expressions), communicative tools (e.g. language, text, and speech) and digital tools (a calculator or computer software). Tools can enable and shape mathematical thinking. They are used for some purpose in order to achieve something (Roth & Radford, 2011).

Developing ML requires positive dispositions toward using mathematics and an appreciation of mathematics and its benefits (Jablonka, 2003). Positive dispositions involve willingness and confidence to engage with mathematics. Figuring out how to solve problems occurring in everyday life requires one to think flexibly about mathematics and adapt the methods and procedures to the current context (De Lange, 2003; Schoenfeld, 2001). Therefore, the competence to think creatively is an important part of life and ML. It involves both taking the risk of not succeeding and the initiative to try different approaches.

All the elements are grounded in a critical orientation. ML is about recognising the power and risk when issues are expressed numerically and to critically consider the contexts, mathematical knowledge and tools involved. Mathematically literate individuals recognise the role mathematics plays in culture and society, for example, how mathematical information and practices can be used to persuade, manipulate, disadvantage or shape opinions about social or political issues (Jablonka, 2003). Hence, they know and can use efficient methods and evaluate the results obtained (Goos et al., 2014).

Teaching and learning mathematics in contexts

Although teachers recognise the contextual and applied aspect of ML (Genc & Erbas, 2019), they count a wide range of practices as real-world connections (Gainsburg, 2008). Teachers make such connections frequently, but the connections are brief and do not require any thinking from the students. Therefore, Gainsburg claims that teachers' main goal is to impart mathematical concepts and skills, and the development of students' competence and disposition to recognise applications and solve real problems is of lower priority. Wijaya et al. (2015) argue that to create opportunities for students to learn to solve contextualised tasks, teachers can ask the students to paraphrase the problem, encourage them to identify the relevant mathematical procedures and verify the reasonableness of the solution.

It is usually expected that students are more interested in contextualised problems. Andersson et al. (2015) report that students experience meaningfulness and engagement when mathematics is related to societal issues and that their engagement in mathematics learning is influenced by experiences related to the task, situation, school organisation and the socio-political. However, if the particular context is of low interest, students are more interested in solving problems without real-life connections (Rellensmann & Schukajlow, 2017). Therefore, various aspects of the context need to be considered. Authentic contexts do not necessarily involve authentic questions that people in the real context would pose or authentic methods that people in the real context would use. For an aspect in education to be considered as authentic, it requires an *out-of-school origin* and a *certification* of provenance either physically or by an expert (Vos, 2018). This means that authenticity should be made explicit to the students. Also, there are different views in the mathematics education community regarding what counts as real. For instance, in realistic mathematics education (RME), a fantasy world can be a suitable context as long as it is real in the student's mind and students can engage productively with mathematics when it is explored in imaginative settings (Nicol & Crespo, 2005).

Hence, students' predispositions to transfer mathematics learning in school to real-life situations are complex and varied because contexts are part of an interaction between students' experiences, goals and perceptions of the mathematical environment (Boaler, 1993). Students' view of mathematics as a school activity and not as a way to make sense of the world creates a dichotomy between everyday mathematics and school mathematics in the sense that formal learning fails to benefit from the intuitive knowledge students bring to the classroom, and students are unable

Table 1 Overview of generated data and the number of participants involved

	Number of observed lessons	Number of students interviewed	Number of students in the class
School A	6	8	14
School B	5	7	28
School C	5	7	18[a]
Total	16	22	60

[a]In school C, the class was divided into two groups according to which students had consented to participate in the research. There were 28 students in total in the class.

to generalise their mathematical knowledge to situations outside school (Hunter et al., 1993). As teachers' ideas for making real-world connections come from their own experiences (Gainsburg, 2008), their understanding of how to apply mathematics in real-world contexts is important for providing students with the learning experiences necessary to adapt the knowledge they learn in school to the outside world (Popovic & Lederman, 2015).

Method

Participants and procedures

Data were generated in three schools in Western Norway. I refer to the schools as A, B and C. The schools' total number of students on roll range from 220 to 370 and all three schools teach grades 1 through 10. The three schools cooperate with the author's former university teacher education programme and were therefore recruited for convenience.

I contacted the school leaders, and they recruited teachers and their respective classes. Criteria for selection of classes were that they were grade 9 (students aged 14–15 years) and that they agreed to participate. I needed consent from both the students and their parents. All parties involved received written information explaining my interest in studying teaching concerning concepts in policy documents. To ensure informed consent, I attended meetings with the teachers, the students, and the parents.

Methods for data generation are interviews and lesson observations. The number of participants involved from each school is displayed in Table 1.

I instructed the teachers to plan and conduct the teaching as they would normally as I was interested to observe, as far as possible, regular mathematics lessons. Therefore, I was not involved in decisions regarding the mathematical topics taught, or the activities worked with in the lessons. In schools A and B, all lessons concerned the topic equations. In school C, the first two lessons concerned equations and the rest concerned percentages. The lessons varied in length from 45 to 90 min.

I was a non-participant observer and did not intervene in the lessons, other than by being present.

Individual semi-structured interviews were conducted with 22 students. To investigate students' encounters with ML, I asked questions about what mathematical knowledge they need and in which contexts they need it. Also, I asked questions about what their parents or other people they know use mathematics for. The belief was that by thinking of someone they know, students would have a starting point for further reflection about the use of mathematics in the real world. I developed an interview guide with questions and topics I wanted them to reflect upon but without a predetermined sequence. Each interview lasted about 15 to 20 min. The interviews were recorded and transcribed.

To capture the classroom activity from the students' perspective, I video recorded 16 mathematics lessons using head-mounted cameras. For each lesson, three different students wore head cameras, recording the classroom activity. Head cameras enabled me to capture the participants' visual fields, get more in-depth insight onto the direction and timing of participant attention and document participant actions. They also provided me with valuable insight into students' conversations, the tasks and students' written accounts and their attention toward the blackboard (or elsewhere), all in one recording.

Process of analysis

The interviews and lesson recordings were loaded into the computer-assisted qualitative data analysis software NVivo. Interviews were transcribed verbatim. To get an overview of the interview data, I constructed tables based on the students' replies to the questions. The frequency of students' examples of different occupations, everyday situations and mathematical topics was recorded. As students' encounters with ML were the topic of study, the interviews and the lesson observations were closely studied and analysed according to the elements of ML. The operationalisations of the elements of ML in the lesson observations and interviews are presented in Table 2.

Findings

In the following, I present the findings from the observed lessons and the findings from the interviews.

Lessons

Most of the tasks in the observed lessons on equations are strictly mathematical and do not involve contexts. However, the students work on a few word problems with contexts from personal and social life. These tasks contain inauthentic questions and solutions methods. An example is the following task from school B, lesson 2:

> *In a pasture, the length is three times the breadth. The perimeter is 240 meters. What is the area of the pasture?*

Table 2 Operationalisations of ML elements

ML elements	Lessons	Interviews
Context	Do tasks involve contexts from personal and social life, work-life or citizenship? Are applications of mathematics in different contexts discussed? Are authentic aspects of the tasks and certifications discussed?	Do students describe different contexts where mathematics is or can be useful? What characterises these contexts?
Mathematical knowledge	Which concepts and procedures are worked with, and how? Are various methods explained and discussed? Are concepts and representations connected? Are solutions justified?	Do students describe mathematical topics, concepts, procedures, and methods that they or others use or might use?
Tools	Are digital tools, representations and models used to solve or model problems? How?	Do students connect using tools to doing mathematics in context? Do students view tools as a mediator of thought?
Dispositions	Are students engaged in tasks and discussions? Do they show curiosity, interest and confidence by engaging in investigations and discussions? How does the teacher motivate and encourage?	What are the students' views of mathematics? Do students see the benefits of mathematical knowledge? Do students see themselves as mathematics learners and users?
Critical orientation	Are the students involved in discussing, questioning, explaining, evaluating, and validating methods and solutions? Is mathematical information used to make decisions and judgements, add support to arguments and challenge an argument or position?	Do students recognise the role mathematics plays in society in general, as a tool to understand, inform, and make judgements? Do students provide examples of situations where they have used, or might use, mathematics to make informed decisions and judgements, or critically evaluate others'?

First, a farmer is unlikely to express the length and breadth of a pasture in terms of an unknown. Second, to find the length and breadth, s/he would go out and measure it. Authentic aspects are not critically discussed in the lessons. Therefore, the tasks do not demonstrate the role mathematics plays in the world. Also, the teachers do not give any certification of contexts where equations are used, even though the students request it. The following excerpt suggests that the students do not see when they would use equations in life outside school, and the teacher cannot provide them with one.

> *Teacher: I remember what you said to me then (in the previous chapter on algebra). "When will we use this?", you said when we worked with all those expressions.*
> *Student: To use it in the next chapter, that was not what we meant. We meant in life. (School B, lesson 2)*

The lessons on percentages in school C all contain task contexts from personal and social life, aimed at showing the use of percentages in the real world. However, these tasks are also traditional word problems and contain inauthentic aspects. Still, in the lessons, the teacher provides some certifications by referring to contexts in real life where knowing percentages are useful. For example, she talks about how some stores advertise discount in terms of money while others use per cent. It is, therefore, useful to calculate percentage in order to evaluate which is the better buy. She also talks about her own experiences when shopping at sales and states:

> *Teacher: There are many things that you learn in mathematics where you ask me "What do we need this for?" But I know from experience that this will be very useful for you later.*

The observed lessons involve great emphasis on developing mathematical knowledge. Conceptual understanding and procedural fluency are emphasised in the sense that students spend most of the time practising the procedures demonstrated on the chalkboard. The procedures concern how to solve linear and quadratic equations, equations with fractions, how to test their solutions, inequalities and word problems. All the observed lessons are organised in similar ways with the teacher demonstrating or explaining a concept or technique on the chalkboard, followed by students working with textbook tasks. Some tasks are solved either by students or by the teacher on the chalkboard. The questions and answers concern carrying out the correct procedure and finding the correct number and do not involve critical discussions about concepts, relationships or alternative solution methods. However, it can be argued that testing a solution is a way of critically evaluating the answer.

The students frequently use calculators to perform calculations. On a few occasions, the teachers use and encourage students to use representational tools. For example, teacher A draws a number line to represent the solution of an inequality, and students are encouraged to make drawings to represent the problems and to mediate their thinking. Also, teacher B emphasises language as an important part of thinking and often tells the students to discuss the methods and strategies with each other or oneself.

Both peer-work and comments about the real-world significance of mathematical knowledge are ways to motivate and engage the students. Also, the teachers try to foster students' positive dispositions and engagement in the tasks through praise and supportive feedback on their work. The tasks worked with do not invite students to be creative and inquire. The students display great varieties in terms of emotions and engagement. Some work concentrated on the tasks throughout the lessons, while others are distracted and unfocused. Some express feelings of enjoyment, while others express dislike.

In terms of critical orientation, there is a lack of critical discussion, justification and evaluation of methods, solutions, concepts and contexts in which they are used. Although methods are the topic of whole-class and peer-group talk, it is, to a large extent, up to the individual to make the critical judgements in is own mind. There is no collective focus on engaging in critical discussions. The goal is to find the correct number, and the contexts (and numbers) are not given any further attention. However, three episodes from the classroom may, to some degree, be related to critical orientation. Two episodes come from the lessons on percentages in school C. One concerns Black Friday sales and evaluating a purchase. The teacher talks about a webpage comparing prices and displaying the price history of different commodities. The second comes from a previous lesson in social sciences where they compared local taxes in neighbouring municipalities and discussed reasons for the large differences. The third comes from school B and concerns equations. The teacher provides the students with a list of points to help them structure word problems and instructs them to read the task carefully, and to look for information not relevant for solving the task:

1. Read the task carefully
2. Find out what they are asking for
3. Find the best point of departure (who/what do know the least about?)
4. Form the equation
5. Check if the equation makes sense
6. Solve the equation to find the unknown
7. See if you have the answer to the task

This list is easily transformed into a general strategy for solving problems and for addressing issues connected to critical orientation such as using mathematics to support an argument. However, the focus is on the equations, and the list's potential for developing critical orientation is not fulfilled. A common feature in all three examples is that they are all led by the teachers and do not involve any student action.

Interviews

In the interviews, the students mentioned 11 different examples of situations from daily life involving mathematical knowledge. In total, there were 45 examples, as some students mentioned the same situations. There were 32 different examples of occupations involving mathematics and a total of 84. The students also connected different mathematical topics and knowledge to everyday and occupational situations. Table 3 shows the number of times different mathematical topics were connected to contexts in everyday life or occupations.

In the interviews, students gave examples of the mathematical knowledge they, their parents or others need in everyday life. Their responses concerned the topics geometry, money and finances, calculations and counting (arithmetic), percentages, measurements, fractions and equations and algebra. The students were unsure about the need for equations. Some students commented that some occupations might require equation solving, but they could not provide an example of what they need equations for. Two students also mentioned digital tools (spreadsheets) as relevant in some occupations. In the interviews, students only commented on specific mathematical topics and did not talk about problem-solving strategies or mathematical skills, except doing mental calculations.

The students connected specific mathematical knowledge and tools to specific contexts. They reflected on situations in which they, their parents or someone they know need to formulate, represent and solve a mathematical problem. The contexts in the students' examples concern personal and social life and work life. The

Table 3 Mathematical topics in everyday life

Mathematical topic	Everyday	Occupations
Geometry (area, length)	14	23
Money and finances	3	8
Calculations and counting (mental arithmetic, the four arithmetic operations)	23	21
Measurements (time, weight, volume)	27	12
Percentages	7	3
Equations and algebra	0	5
Fractions	1	1

students commented that mathematics is necessary to manage personal finances, i.e. to pay bills, plan what to spend money on, and "At the store, if I am buying several things, to calculate how much it costs" (Student in school B). Mathematics is also required when cooking, planning a journey or redecorating the house. For example, a student in school C commented: "Not long ago, I wanted to buy a new desk, and then I had to measure my room to find out if it would fit or not". The students relate mathematical knowledge to performing basic procedures aimed at producing a specific number. Some students relate mathematics to school. For example, a student in school A said: "I use it for homework and stuff, of course. And here." Relating to progression in upper secondary school, another student in school A said: "You have to learn in mathematics to get further." This could indicate that students see mathematics as relevant for their further education in terms of admission to schools and further studies. Also, they spend a big part of their day in school and therefore connected their use of mathematics in everyday life to school. On the other hand, it may display a view of mathematics as something detached from life outside school.

From contexts in work life, students referred to different occupations and examples of mathematics needed by professionals in their work. Carpenters need knowledge about mathematics in order to build houses correctly, for example, find the area of the rooms or "to measure how long that plank has to be" (Student in school B). Shop assistants need to do mental arithmetic and percentages to calculate prices and sums of commodities. The students also commented that doctors and nurses use mathematics for calculations so that the patients get the right medicine dosage. Leaders and economists need mathematics to deal with budgets, salaries and purchases. The students believed that mathematics is needed in most occupations. No one was able to give examples of occupations where mathematics is not needed. However, they believed that some occupations require more mathematics than others, or as a student in school C stated, "It is smart to know maths either way".

The students did not give any examples of mathematics used in contexts concerning citizenship or societal issues, which suggests that they have not had sufficient encounters with ML in such contexts. Societal issues are important in the development of ethical and reflective subjects. The contexts students mention are contexts that are certified, either by parents or relatives or by their own experiences.

The fact that all students were able to give examples of how mathematics is used in the real world suggests that they, at least to some extent, appreciate the role mathematics plays in the world and as such hold positive dispositions toward mathematics. Some students express that mathematics is difficult and that they do not think they use it often. Still, they acknowledge that there may be situations where they are involved in mathematical activity without reflecting upon it. One can argue that in such situations, they use mathematics that they have encountered several times and have become part of them. On the other hand, it might be that the mathematics involved has not yet become part of their consciousness.

The interviews contain little evidence of a critical orientation. Although students can recognise some of the role mathematics plays in specific contexts, they do not comment on how mathematics is used to form an argument or justify a position. Students have a narrow view of mathematics as numbers, calculations (the four arithmetic operations) and a way to find solutions. A few students relate these solutions to problems in everyday life, such as shopping and cooking. Mathematics is related to practising procedures and performing calculations, and not as a way to make sense of the world.

Discussion

In ML, context is the central element, but from the observed lessons and interviews, formal mathematical knowledge seems to be central. Although teachers believe that they are making mathematics relevant to the students by offering contextualised tasks, they may be reinforcing students' narrow view of the subject and ML by only considering the importance of the mathematical topic and not the significance or authenticity of the contexts and tasks and their potential to teach about the context (Gainsburg, 2008). There is a lack of certifications and critical discussions about context, mathematical knowledge and tools in the lessons. This may contribute to the narrow view of mathematics displayed in the interviews.

Some points in the list provided by teacher B can be related to Wijaya et al.'s (2015) framework for teaching practice supportive for students' opportunities to solve contextualised tasks. However, the list involves specific references to using equations, which do not encourage students to explore various procedures to solve the problem. It may support a view of mathematical problems as having only one approach and one solution (Vos, 2018). Also, the list is used for solving traditional word problems where there is, in fact, a preferred procedure and a fixed solution. If presented in a general way, the list might help students develop strategies for solving all kinds of problems in which they initially do not know how to solve, and in that way might contribute to developing students' ML.

According to TO, mathematics education is a cultural, political and societal endeavor (Radford, 2016). Nosrati and Andrews (2017) express disappointment in that the students in their study did not see mathematics as a cultural artefact or as an education for citizenship. From the observed lessons reported here, such views of mathematics could not be expected. Research has shown that teachers struggle to implement authentic and meaningful contexts and activities involving citizenship (Goos et al., 2014). This seems to be the case in the observed lessons as well. Therefore, if the students have not encountered citizenship and cultural issues in the mathematics classroom, how can we expect them to be part of their consciousness (Radford, 2015)? The interviews show that although students are conscious of the use of mathematics in several contexts, this consciousness is confined to very basic mathematical operations performed in word-problem–like contexts. This resonates with the findings of Nosrati and Andrews (2017). If these findings are prevailing in other classrooms as well, we are currently not preparing students for the demands of the twenty-first-century workplace and world (Gravemeijer et al., 2017).

Manifestations of mathematical illiteracy are prevalent in society, for example, in terms of mathematical errors in newspapers (De Lange, 2003). Either the content of mathematics learned in school is not making citizens mathematically literate or the structural design of teaching practices is not helping students make connections to real-life situations. From the results reported here, I argue that the problem lies with the teaching practices. Although ML has been considered a basic competence in Norway since 2006, and problem-solving and real-world connections even longer, it appears that teaching is still following the findings of Wijaya et al. (2015), where teachers mainly used a direct instructional approach and reflected a mechanistic view of school mathematics as pure mathematics and context-based tasks as plain word problems. If teaching practice fails to involve students in posing and answering questions, making inquiries and solving open-ended problems, students will continue to view mathematics only as a school activity, and the contexts to which students relate the use of mathematics will continue to be limited to basic everyday activities. The social justification of mathematics depends on its potential use in real-life situations. For individuals to develop their ML learning and becoming, they need to encounter the use of mathematics in real-life situations a sufficient number of times, and the situations need to be significant to the students. According to Mellin-Olsen (1981), "the determination of this 'sufficient number' and of the significant situations is, of course, the difficult crux of our problem, where we have to focus our energies when preparing practice". As students still hold the view of mathematics as detached from the reality outside school, and teaching still supports this view, it seems like this crux is just as challenging 40 years later.

Not every mathematical topic that students learn in school has an apparent application in their daily lives. The application of equations and algebra seems to be particularly challenging to demonstrate. Equations may, therefore, not be the best-suited topic of study when investigating students' encounters with ML. However, this was the topic at the time of my visits. On the other hand, the issues arising from the analysis have a didactical dimension that goes beyond the specific mathematical topic. In abstract mathematical topics, such as equation solving, there are opportunities to engage in the mathematical tasks in ways that are aligned to an education focused on ML. Such opportunities could be taken, for example, by focusing on developing positive dispositions, the use of different tools, and developing a critical orientation toward the procedures and answers. Further research should focus on how teaching can provide students with encounters of mathematics in real life to support their objectification of ML, for example through tasks involving learning about both context and mathematical topic, such as mathematical modelling tasks (Steen et al., 2007; Vos, 2018). Research on how a critical orientation can be implemented in teaching in all school levels is of great importance.

In this study, ML is framed within the perspective of TO. The tasks and examples in the observed lessons and interviews are actualisations of the potential knowledge of ML. The teachers' and students' thoughts and actions are a result of cultural and historical ways of thinking and doing. Such cultural and historical ways of thinking and doing characterise students' encounters with ML. These encounters concern developing mathematical knowledge for personal advancement (Nosrati & Andrews, 2017) instead of becoming ethical and reflexive subjects in the world (Radford, 2016), and they are also results of our history and culture. I believe that interpreting ML in terms of TO can provide a new perspective on how ML can be understood and developed. This perspective should be further explored.

Funding Open access funding provided by Volda University College.

Declarations

Ethical Approval All procedures performed in the study involving human participants were in accordance with the ethical standards of the institutional and national research ethics committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards. Informed consent was obtained from all individual participants involved in the study.

Conflict of Interest The author declares that there is no conflict of interest.

References

Andersson, A., Valero, P., & Meaney, T. (2015). "I am [not always] a maths hater": shifting students' identity narratives in context. *Educational Studies in Mathematics, 90*(2), 143–161. https://doi.org/10.1007/s10649-015-9617-z

Boaler, J. (1993). Encouraging the transfer of "school" mathematics to the "real world" through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*(4), 341–373. https://doi.org/10.1007/BF01273906

D'Ambrosio, U. (2007). The role of mathematics in educational systems. *ZDM Mathematics Education, 39*, 173–181. https://doi.org/10.1007/s11858-006-0012-1

De Lange, J. (2003). Mathematics for literacy. In B. L. Madison, & L. A. Steen (Eds.), *Quantitative literacy. Why numeracy matters for schools and colleges* (pp. 75–89). Princeton: The National Council on Education and the Disciplines.

Frankenstein, M. (2010). Developing a criticalmathematical numeracy through real real-life word problems. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference* (Vol. 1, pp. 248–258). Berlin: Freie Universität Berlin.

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education, 11*(3), 199–219. https://doi.org/10.1007/s10857-007-9070-8

Geiger, V., Forgasz, H., & Goos, M. (2015). A critical orientation to numeracy across the curriculum. *ZDM Mathematics Education, 47*(4), 611–624. https://doi.org/10.1007/s11858-014-0648-1

Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: a survey of the state of the field. *ZDM Mathematics Education, 47*(4), 531–548. https://doi.org/10.1007/s11858-015-0708-1

Genc, M., & Erbas, A. K. (2019). Secondary mathematics teachers' conceptions of mathematical literacy. *International Journal of Education in Mathematics, Science and Technology, 7*(3), 222–237.

Goos, M., Geiger, V., & Dole, S. (2010). Auditing the numeracy demands of the middle years curriculum. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education. Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 210–217). Freemantle: MERGA.

Goos, M., Geiger, V., & Dole, S. (2014). Transforming professional practice in numeracy teaching. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming mathematics instruction: Multiple approaches and practices* (pp. 81–102). Springer International Publishing.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education, 15*(1), 105–123. https://doi.org/10.1007/s10763-017-9814-6

Hunter, J., Turner, I., Russell, C., Trew, K., & Curry, C. (1993). Mathematics and the real world. *British Educational Research Journal, 19*(1), 17–26. https://doi.org/10.1080/0141192930190102

Jablonka, E. (2003). Mathematical literacy. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education* (pp. 75–102, Springer International Handbooks of Education, vol. 10). Dordrecht: Kluwer Academic Publishers.

Liljedahl, P. (2015). Numeracy task design: a case of changing mathematics teaching practice. *ZDM Mathematics Education, 47*(4), 625–637. https://doi.org/10.1007/s11858-015-0703-6

Mellin-Olsen, S. (1981). Instrumentalism as an educational concept. *Educational Studies in Mathematics, 12*(3), 351–367.

Nicol, C., & Crespo, S. (2005). Exploring mathematics in imaginative places: rethinking what counts as meaningful contexts for learning mathematics. *School Science and Mathematics, 105*(5), 240–251. https://doi.org/10.1111/j.1949-8594.2005.tb18164.x

Niss, M., & Jablonka, E. (2014). Mathematical literacy. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 391–396). Springer.

Nosrati, M., & Andrews, P. (2017). *Ten years of mathematics education: preparing for the supermarket?* Paper presented at the CERME 10, Dublin, Ireland, 2017–02–01.

Noss, R., Hoyles, C., & Pozzi, S. (2000). Working knowledge: mathematics in use. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 17–35). Kluwer.

OECD (2012). PISA 2012 assessment and analytical framework. Mathematics, reading, science, problem solving and financial literacy.

Olsen, R. V. (2006). A Nordic profile of mathematics achievement: myth or reality? In J. Mejding, & A. Roe (Eds.), *Northern lights on PISA 2003: A reflection from the Nordic countries* (pp. 33–45, TemaNord). København: Nordic Council of Ministers.

Popovic, G., & Lederman, J. S. (2015). Implications of informal education experiences for mathematics teachers' ability to make connections beyond formal classroom. *School Science and Mathematics, 115*(3), 129–140. https://doi.org/10.1111/ssm.12114

Radford, L. (2015). The epistemological foundations of the theory of objectification. *Isonomia - Epistemologica, 7*, 127–149.

Radford, L. (2016). The theory of objectification and its place among sociocultural research in mathematics education. *International Journal for Research in Mathematics Education, 6*(2), 187–206.

Rellensmann, J., & Schukajlow, S. (2017). Does students' interest in a mathematical problem depend on the problem's connection to reality? An analysis of students' interest and pre-service teachers' judgments of students' interest in problems with and without a connection to reality. *ZDM Mathematics Education, 49*(3), 367–378. https://doi.org/10.1007/s11858-016-0819-3

Roth, W.-M., & Radford, L. (2011). *Cultural-historical perspective on mathematics teaching and learning*. Springer.

Schoenfeld, A. H. (2001). Reflections on an impoverished education. In L. A. Steen (Ed.), *Mathematics and democracy: The case for quantitative literacy* (pp. 49–54). Princeton.

Sfard, A. (2008). Thinking as communicating. Human development, the growth of discourses, and mathematizing. Cambridge: Cambridge University Press.

Skovsmose, O. (2011). *An invitation to critical mathematics education*. SensePublishers.

Stacey, K., & Turner, R. (Eds.). (2015). *Assessing mathematical literacy: The PISA experience*. Springer International Publishing.

Steen, L. A. (Ed.). (2001). *Mathematics and democracy: the case for quantitative literacy*. Princeton.

Steen, L. A., Turner, R., & Burkhardt, H. (2007). Developing mathematical literacy. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 285–294). Springer.

The Norwegian Directorate for Education and Training. (2012). *Framework for basic skills*. Ministry of Education and Research.

Vos, P. (2018). "How real people really need mathematics in the real world" - authenticity in mathematics education. *Education Sciences, 8*(4), 195–208.

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Teachers' teaching practices and beliefs regarding context-based tasks and their relation with students' difficulties in solving these tasks. *Mathematics Education Research Journal, 27*(4), 637–662. https://doi.org/10.1007/s13394-015-0157-8